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COMMENT

Comment on 'Eden model on the Manhattan lattice'

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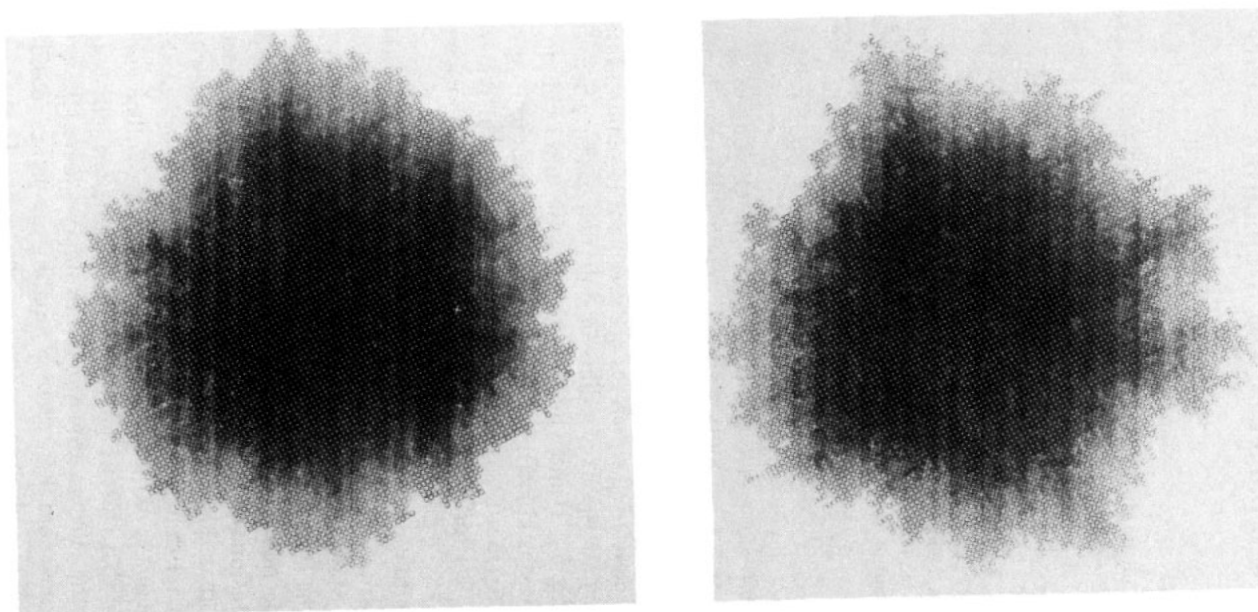
Abstract. Numerical simulations of the Eden model on the Manhattan lattice show clearly that this model leads to compact objects similar to the standard Eden case. This contradicts the conclusion of a letter of Chernoutsan and Milošević.

By applying a position space renormalisation group technique to the Eden model on square Manhattan lattice, Chernoutsan and Milošević (1985) concluded that this model and the Eden model on the ordinary square lattice were in two different universality classes. Moreover, they argue that the former may have a non-trivial fractal dimension. Unfortunately, renormalisation equations are very hard to obtain exactly. In practice, the authors can calculate the fixed points in only two cases: when the renormalised cell is 2×2 (linear size: $b=2$) and when it is 3×3 ($b=3$). For example, in the extended Eden model one guesses compact clusters in any dimension of space (Richardson 1973, Peters *et al* 1979, Dhar 1985), and the authors find a 'fractal' dimension: $D=1.778$ for $b=2$ and $D=1.7991$ for $b=3$, results which show a very poor convergence to the expected value $D=2$.

To enlighten the situation, it is interesting (as challenged by the authors) to compare these conclusions with numerical results. I did numerical simulations of the Eden model on the square Manhattan lattice. The results for cluster size up to 50 000 particles are plain. Figure 1(b) shows a typical cluster grown by this process. Figure 2 shows a log-log plot of the radius of gyration against the number N of particles and an effective fractal dimension (defined as $(d\log(R_G)/d\log(N))^{-1}$) against $1/N$. We know (Peters *et al* 1979) that this simple method gives good results, when we are only interested in the fractal dimension of the whole cluster (it is not the case for such quantities like the thickness of the surface; for example, see Jullien and Botet (1985a)).

It appears clearly that clusters grown on the Manhattan lattice are compact and look very much like the clusters grown on an ordinary square lattice (figure 1(a)).

Nevertheless, we can note that, when comparing the values found for the 'fractal' dimensions by the renormalisation technique, one finds: $D_E \leqslant D_{EM} \leqslant D_{EE} \leqslant 2$ (the values are: $1.729 < 1.7302 < 1.7991 < 2$ for $b=3$), where index E means the standard Eden model (the A model of Jullien and Botet (1985a, b)), EE the extended Eden model (B version) and EM the extended model on Manhattan lattice. Then though all these dimensions are probably equal (to 2), the increasing order in D seems to correspond to a decreasing order in the width of the surface. We can qualitatively see



(a) (b)

Figure 1. (*a*) Typical cluster of about 8000 particles grown by the Eden process on an ordinary square lattice. The different grey ton s i dicate the order of growth. The later a particle arrives the lighter its colour; (*b*) same for a Manhattan square lattice.

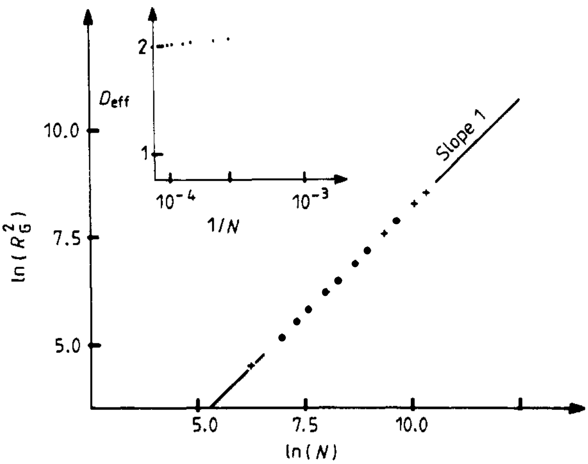


Figure 2. Log-log plot of the radius of gyration against the number of particles for the Eden process on the Manhattan lattice (average over 250 samples). Insert: effective fractal dimension (see text) against $1/N$.

this feature in figure 1 for models EM and EE. It is reminiscent of the non-renormalisable constraint due to the Manhattan lattice (short range effect). This constraint makes the surface rougher on the short length scales.

Moreover, the method used by Chernoutsan and Miloševič is not credible because the renormalised cells are too small. To give an example: for the extended Eden model, they found for the critical value of the fugacity: $K_c = 0.377$ for $b = 2$ and $K_c = 0.231$ for $b = 3$. What happens for large values of b?

For every rescaling factor b, the renormalisation group transformation is

$$K' = R(K)$$

where $R(K) = a_{2b-1}(b)K^{2b-1} + \ldots + a_{b^2}(b)K^{b^2}$, and each coefficient $a_i \geqslant 1$ since it is a number of configurations. First we note that the existence of a unique unstable fixed point in the range $0 \leqslant K \leqslant 1$ is a trivial result. Then, a lower bound of a_{b^2} is easy to obtain, since the number of ways to construct the full $b \times b$ cell is larger than the number of ways to construct the full $(b-1) \times (b-1)$ cell times the number of ways to fill the border (figure 3).

So, we have the inequality

$$a_{b^2}(b) > (2b-2)!^2 a_{(b-1)^2}(b-1).$$

Since $\Pi_{c=0}^{b-1} (2c)!^2$ is of order b^{2b^2} when b tends to ∞, the inequality

$$K_c > a_{b^2}(b) K_c^{b^2}$$

implies

$$K_c \leqslant b^{-2}$$

for b tending to ∞ (for any d, one has: $K_c \leqslant b^{-d(d-1)}$).

This result shows that the limiting value of K_c is 0 and there is no more unstable fixed point when b is infinite. The system does not present any criticality.

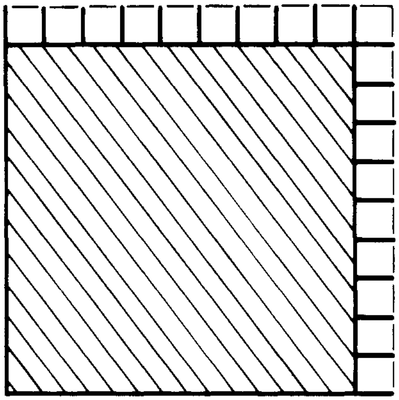


Figure 3. For filling the $b \times b$ cell, one can fill the $(b-1) \times (b-1)$ cell (shaded area) then one fills the heavy links $(2b-2)!$ number of ways; finally one fills the light links $(2b-2)!$ number of ways.

In conclusion: numerical simulations suggest that the Eden model on the ordinary square lattice and the Eden model on the square Manhattan lattice are very similar and probably in the same universality class, in contradiction with the conclusions of Chernoutsan and Milošević based on renormalisation of small blocks. The two models lead to compact objects and the short range constraint due to the Manhattan lattice is not renormalisable.

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Note added. Chernoutsan and Milošević give the example of the Sawada model as a modified Eden model with short range constraint, which yields clusters with non-trivial fractal dimension. This result has been quashed by Meakin (1983).

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